Oleksandr NESTORENKO
Associate Professor, PhD in Economics, Berdyansk Institute of State and Municipal Management of the Classic Private University, Berdyansk, Ukraine, 3 Portova, buld.2, *e-mail:anestorenko@mail.ru*

INVENTORY ANALYSIS MODELS WITH SHORTAGES WITH INDEPENDENT OF TIME WAITING SANCTIONS IN THE SYSTEM OF „SELLER-BUYER"[1]

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ABSTRACT

The optimal parameters of functioning of a seller's logistic system do not always satisfy the requirements of a buyer. This situation often occurs when there's the task of inventory analysis models with the compensated shortages. The optimal time for the absence of goods from the perspective of the seller may not be large enough. The penalties for waiting of the goods by the buyer do not compensate his loss during this time. Building a model of inventory management with the requirements of the buyer allows optimizing the logistics processes of the seller.

KEYWORDS

optimization, economic-mathematical models, logistic processes

INTRODUCTION

Economic and mathematical models of inventory analysis with shortages are a generalization of the model of Wilson (EOQ model). They are divided into two types – with uncompensated and compensated shortages. These models have a common ideology of building and the same mathematical formalization [1]. The dependence of total costs TC for time T on the time of availability of goods t_1 and time between deliveries t_s is:

$$TC(t_1,t_2) = \frac{c_s T}{t_s} + \frac{1}{2} c_1 \mu T \frac{t_1^2}{t_s} + \frac{1}{2} c_2 \mu T \frac{(t_s - t_1)^2}{t_s},$$

where c_s– delivery charges of one consignment, c_1– cost of storage of one goods' unit, c_2– penalties from the emergence of a unit of goods' shortage, μ– daily demand.

The differences in these models are in understanding the penalties for shortages.
In the case of uncompensated shortages under penalties the entire amount of the money which was not received from the sale of units of goods is meant, i.e.the goods are purchased at a price *p*, in case of theirpresence are sold at a price (1+*R*)*p*, in case of the absence of goods –the non-receipt amount isc_2=R*p*.
In a situation with the compensated shortage under penalties the fee for waiting goods

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(discount) is meant, i.e. goods are purchased at a price p, in case of their presence are sold at price $(1+R)p$,in case of their absence are sold at the time of the goods' delivery at the price $(1+R_1)p$, i.e. the discount is $c_2=(R-R_1)p$.

The analysis of literary sources on the topic of „Inventory Analysis" has determined the problem: in the optimization of real logistic processes the models of inventory control with shortages are relatively seldom used, as the simulation results differ significantly from the results obtained in practice, recommendations for management decisions grounded by these models, often contradict the logic of economic processes. Therefore, these models are mostly regarded as theoretical and inconvenient to practical activities. In the specialized literature there are some explanations for this problem. So in works [2-4] the main causes of this problem are considered:
- models of inventory management simplistically describe the real economic processes and do not take into account many of the factors influencing them;
- when using the models in practice, their parameters are replaced by estimates, which can differ significantly from their values.
The combination of these and other reasons, according to experts, is the cause for the differences between the theoretical and practical results in inventory management.
Some researchers have remarks to the process of constructing economic and mathematical models of inventory analysis:
- in the models the sums of money belonging to different moments of time are not given to a single point [5, 6];
- in the model with uncompensated shortages, the shortages must be calculated as the product of the daily demand during the absence of the product $\mu(t_s- t_1)$, but not cumulatively as in (1) $\frac{1}{2}\mu(t_s - t_1)^2$ [7];
- in the model with compensated shortages in the case of a decision about the discount that does not depend on the waiting time, the size of the shortages should also be calculated as the product of the daily demand during the absence of the product, the cumulative approach is used in the case of a decision on the discount amount as payment for the expectation of the product for one day [7].
Models of inventory control with shortages [1] allow optimizing the logistics processes of the seller. However, they do not take into account the requirements of buyers. Quite often, the optimal parameters of the logistic flows of the seller are unacceptable to the buyer.
The construction of economic and mathematical models of inventory management with consideration of the mentioned observations will improve the adequacy and increase the effectiveness of their practical application.

Model of inventory control with shortages without taking into account the requirements of buyers

Taking into account the comments to the building process, the model of inventory management with uncompensated shortages has the form [7]:

$$PR(t_1,t_S) = \left(\frac{(1+R)p\mu}{\ln(1+r)}((1+r)^{t_1}-1) - (c_S + p\mu t_1)(1+r)^{t_1}\right)\frac{((1+r)^T-1)}{(1+r)^{t_S}-1},$$

where $PR(t_1, t_S)$ – the profit of the seller for the period T, r is the relative interest rate per day.
Function (2) reaches its maximum when $t_1= t_S= t_{SW}$, where t_{SW} is the optimal value defined by the formula of Wilson:

$$t_{SW} = \sqrt{\frac{2c_S}{rp\mu}}.$$

So for the seller the work with uncompensated shortages is economically inefficient. In this case the buyer does not have a question about waiting for the product.

Taking into account the comments to the building process, the model of inventory control with compensated shortages and penalties, independent of the waiting time has the form [7]:

$$PR(t_1, t_s) = \left(\frac{(1+R)p\mu}{\ln(1+r)}((1+r)^{t_1} - 1) - (c_s + (1+R_1)p\mu t_1 - R_1 p\mu t_s)(1+r)^{t_1}\right).$$

When $R - R_1 \geq (1 + r)^{t_{SW}} - 1$ the function (4) reaches a maximum value at $t_1 = t_S = t_{SW}$ i.e. the shortages are not allowed.
Otherwise, there exists an optimal solution allowing working with the compensated shortages. It is found from the system of nonlinear equations:

$$\begin{cases} x = R - R_1 - \frac{1}{2}z_w^2 + R_1 y \\ E^x = 1 + R - R_1 E^{-y} \end{cases},$$

$$\text{where} z_w = t_{sw}\ln(1+r), x = t_1\ln(1+r), y = (t_s - t_1)\ln(1+r),\ z = t_s\ln(1+r).$$

When „small" x and y, the solution of the system (5) has the form:

$$t_{20} = t_{SW}\frac{\sqrt{\frac{1+R_1-A^2}{R_1}} - A}{1+R_1}, \quad t_{10} = t_{SW}\frac{A + \sqrt{R_1}\sqrt{1+R_1-A^2}}{1+R_1}, \quad t_{S0} = t_{SW}\sqrt{\frac{1+R_1-A^2}{R_1}},$$

$$\text{where } A = \frac{1}{z_W}\left((R - R_1) - \frac{1}{2}z_W^2\right), t_2 - \text{absence of goods in stock.}$$

Let us consider the dependence of the optimal time for absence of goods t_{20} on the size of the discounts (Fig. 1).

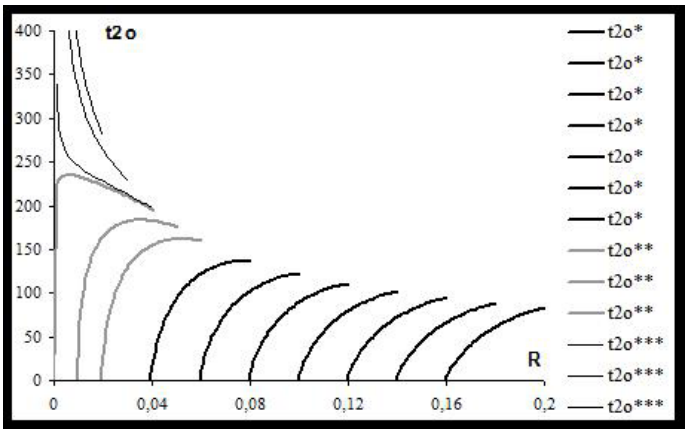

Fig. 1. The dependence of the optimal time of the product's absence on the amount of discount
Rys. 1. Zależność optymalnego czasu braku towaru od rozmiaru zniżek

When „big" R the optimal time for the absence of goods t_{20}^{*} increases with a decrease in the discount rate, with R approaching the value equal to $(1 + r)^{t_{sw}} - 1$, the optimal time for the absence of product t^{20**} at first increases, then decreases with decreasing of the size of discount, if $R \leq (1 + r)^{t_{sw}} - 1$, the optimal time for the absence of goods t^{20***} decreases with decreasing size of the discount.
If the size of the discount $(R - R_1)p$ over time of expectation of the product t_{20} satisfies the customer, the optimal parameters of the logistic process with the compensated shortages are determined by formulas (6). In this case, the seller receives the maximum profit, equal to:

$$\mathrm{PR}_0 = \mathrm{PR}(t_{10}) = \frac{((1+r)^T - 1)p\mu}{r}(R - ((1+\mathrm{r})^{t_{10}} - 1)).$$

Model of inventory management with the shortages, taking into account the requirements of buyers

The period of goods' absence may be large enough, which will lead to the failure of the buyer to expect the product at a proposed discount. In [8] the terms of decision-making on purchase of the goods by the customer are defined. If the customer has decided to buy, he will agree to expect the goods at time t_k under the condition of exceeding the size of the discount over losses (lost profit) from the absence of product during this time. This condition is equivalent to the following:

$$t_k \leq \frac{\ln\left(1 + \frac{(R-R_1)r_k p}{K-(1+R)r_k p}\right)}{\ln(1+r_k)} \approx \frac{(R-R_1)p}{K-(1+R)r_k p},$$

where K – losses (lost profit) from the absence of goods for the day, r_k is the relative interest rate a day for the buyer.

In Fig. 2 dependences of the optimal time for the absence of product t_2 from the solution (6), and the maximum waiting time of a customer t_k from inequality (8), on the size of discounts at different R $(R^{*} > R^{**} > R^{***} > R^{****} > R^{*****})$ are compared.

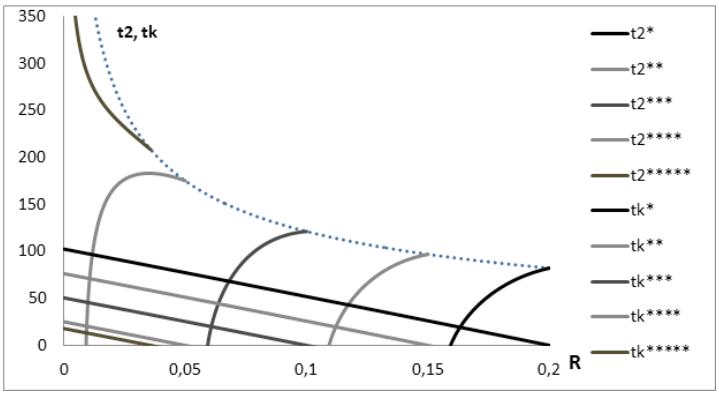

Fig. 2. The dependence of the optimal time of the product's absence t_2 and the maximum waiting time of customer t_k on the amount of discount
Rys. 2. Zależność optymalnego czasu braku towaru t_2 i maksymalnego czasu oczekiwania klienta t_k od rozmiaru zniżek

If the optimal time for the absence of product t_2 is smaller than the maximum waiting time of the client t_k, then the optimal solution is $t_1^- = t_{10}$, $t_2^- = t_{20}$ (Fig. 3, 4), which is found in formulas (6). The maximum profit is calculated according to the formula (7).

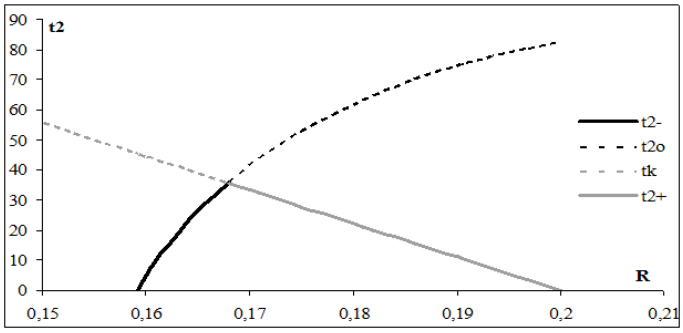

Fig. 3. The optimum values of time for the product absence t_2 in the „seller-buyer" system
Rys. 3. Optymalny czas braku towaru t_2 w systemie «nabywca-sprzedawca»

If the optimal time for the absence of product t_2 is greater than the maximum time of the client's waiting t_k, then the optimal time of the product's absence will be equal to the maximum waiting time of the client $t_2^+ = t_k$, (Fig. 3), and the optimal time of availability of goods in stock t_1^+ is found in the task of maximizing the profit function (9) (Fig.4).

$$PR(t_1) = \left(\frac{(1+R)p\mu}{\ln(1+r)}((1+r)^{t_1} - 1) - (c_s + p\mu t_1 - R_1 p\mu t_k)(1+r)^{t_1}\right)\frac{(1+r)^T - 1}{(1+r)^{t_1+t_k} - 1}$$

The optimum time of goods' availability in stock is:

$$t_1^+ = \sqrt{\frac{2(R-R_1)}{r}t_k + (1+R)(t_k)^2 + t_{SW}^2} - t_k.$$

The maximum profit will be:

$$PR_0 = PR(t_1^+) = \frac{p\mu\left((1+r)^T - 1\right)}{\ln(1+r)}\left(R - ((1+r)^{t_1^+} - 1)\right).$$

The maximum value of the function (7), (11) is reached at minimum value t_{10}, t_1^+. I.e. there is a discount at which the profit reaches its maximum.

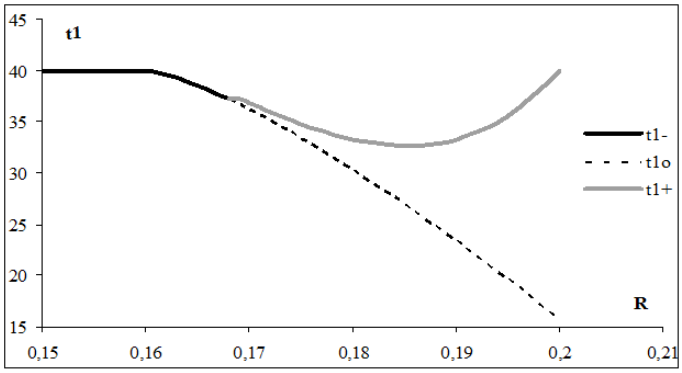

Fig. 4. The optimum values of time of goods' availabilityt_1 in the „seller-buyer" system
Rys.4. Optymalny czas dostępności towaru t_1 w systeme «nabywca-sprzedawca»

When $R_1 = R - ((1 + r)^{t_{sw}} - 1)$ $t_1^- = t_{SW}$, $t_2^- = 0$. At the same time, with the growth of R_1 a function of $t_1^-(R_1)$ decreases.
When $R_1 = R$, $t_1^+ = t_{SW}$, $t_2^+ = 0$.
Therefore, on the interval $R - ((1 + r)^{t_{sw}} - 1) \le R_1 \le R$ there is a minimum of the function $t_1^+ + (R_1)$ in which the profit function (11) reaches its maximum (Fig. 4).
The minimum of the function $t_1^+ + (R_1)$can be located either at the point $t_1^- = t_1^+$ or be a solution to the problem:

$$\begin{cases} t_1^+ = \sqrt{\frac{2(R-R_1)}{r} t_k + (1+R)(t_k)^2 + t_{SW}^2} - t_k \to min \\ t_k = \frac{\ln\left(1+\frac{(R-R_1)r_k p}{K-(1+R)r_k p}\right)}{\ln(1+r_k)} \approx \frac{(R-R_1)p}{K-(1+R)r_k p} \end{cases}.$$

The dependence of the optimal profit on the size is shown in Fig. 5.

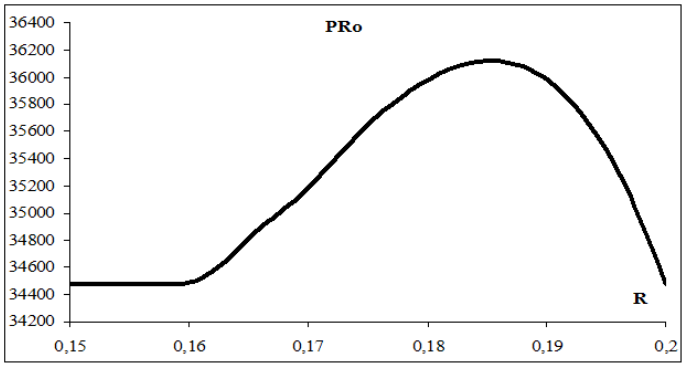

Fig. 5. The dependence of the optimal profit on the seller's discount for the buyer's waiting
Rys. 5. Zależność optymalnego zysku sprzedawcy od wielkości zniżki za oczekiwanie na towar przez nabywcę

Thus, when making policy decisions about a permanent discount for the waiting of the product, the optimal parameters of the logistic process are determined, and while the removal of limitations on the specific discount, the optimal size of discounts that does not depend on the waiting time is also determined.

Model example.
The demand for the products is uniformly distributed for T = 360 days with an averagedaily demand of μ=25 (units)/(days), the cost of delivery of the consignment c_S = 400 €, the purchase price p = 20 €/unit, selling price of 24 € (R = 0,2), the interest rate for the seller is 0,1% per day (r=0,001).
In the absence of goods the buyer bears daily losses in the amount K = 0,03 €, the interest rate for the buyer is 0,05% per day (r_k=0,0005).

Variant 1. The seller makes a decision: shortages are not allowed.

$$t_{SW} = \sqrt{\frac{2*400}{0{,}001*20*25}} = 40\text{days}.$$

After 40 days it's necessary to bring $q_0 = \mu t_{SW}$ = 25*40=1000 units, in this case a profit of 360 days will be

$$\mathrm{PR}_0 = \frac{((1+r)^T-1)p\mu}{r}(R-((1+\mathrm{r})^{t_{SW}}-1)) = \frac{(1{,}001^{360}-1)*20*25}{0{,}001}(0{,}2-(1{,}001^{40}-1)) = 34475\,€.$$

Variant 2. The seller decides: in case of shortages the sale price of goods is 22 € (R_1= 0,1). As 0,1 < 0,2 - (1,001^{40} - 1) = 0,159, to work with the compensated shortages is not economically efficient. Variant 1 is implemented.

Variant 3. The seller decides: in case of shortages the sale price of the goods is 23,3 € (R_1= 0,165).

As 0,165 > 0,2 - (1,001^{40} - 1) = 0,159, to work with the compensated shortages is economically efficient.

$$t_{20} = t_{SW}\frac{\sqrt{\frac{1+R_1-A^2}{R_1}}-A}{1+R_1} = 22\text{ days}, t_{10} = t_{SW}\frac{A+\sqrt{R_1}\sqrt{1+R_1-A^2}}{1+R_1} = 38\text{ days}, \quad t_{S0} = t_{SW}\sqrt{\frac{1+R_1-A^2}{R_1}} = 60\text{ days}.$$

When discount 24 € - 23,3 € = 0,7 € the buyer is willing to wait a maximum of t_k= $\frac{\ln(1+\frac{(R-R_1)r_k p}{K-(1+R)r_k p})}{\ln(1+r_k)}$ = 39days that there is more than the time for the absence of product t_{20} = 22 days..

After 60 days, it's necessary to bring $q_0 = \mu t_{SW}$ = 25*60=1500 units, $q_{10} = \mu t_{10}$ = 25*38 = 950units of product arrive at the warehouse, $q_{20} = \mu t_{20}$ = 25*22 = 550 units are sold at a price of 23,3 € on the day of delivery, a profit of 360 days will be

$$\mathrm{PR}_0 = \frac{((1+r)^T-1)p\mu}{r}(R-((1+\mathrm{r})^{t_{10}}-1)) = \frac{(1{,}001^{360}-1)*20*25}{0{,}001}(0{,}2-(1{,}001^{38}-1)) = 34802\,€.$$

Variant 4. The seller decides: in case of shortages the sale price of the goods is 23,6 € (R_1 = 0,18).

As 0,18 > 0,2 - (1,001^{40} - 1) = 0,159, the work with the compensated shortages is cost -effective.

$$t_{20} = t_{SW}\frac{\sqrt{\frac{1+R_1-A^2}{R_1}}-A}{1+R_1} = 62\text{ days}, t_{10} = t_{SW}\frac{A+\sqrt{R_1}\sqrt{1+R_1-A^2}}{1+R_1} = 30\text{ days}, \quad t_{S0} = t_{SW}\sqrt{\frac{1+R_1-A^2}{R_1}} = 92\text{ days}.$$

When the discount is24€ - 23,6€ = 0,4€ the buyer is willing to wait a maximum of t_k= $\frac{\ln(1+\frac{(R-R_1)r_k p}{K-(1+R)r_k p})}{\ln(1+r_k)}$ = 22 days which is less time for the absence of product t_20=62 days.

$$t_{20} = t_k = 22\text{ days}, t_1^+ = \sqrt{\frac{2(R-R_1)}{r}t_k + (1+R)(t_k)^2 + t_{SW}^2} - t_k = 33\text{ days}, \quad t_{S0} = t_{20} + t_1^+ = 55\text{ days}$$

After 55 days it's necessary to bring $q_0 = \mu t_{SW}$ = 25*55 = 1375 units, $q_{10} = \mu t_1^+$ = 25*33 = 825units arrive at the warehouse, $q_{20}=\mu t_{20}$ = 25*22 = 550 units are sold at a price of 23,6 € on the day of delivery, a profit of 360 days will be

$$\mathrm{PR}_0 = \frac{((1+r)^T-1)p\mu}{r}(R-((1+\mathrm{r})^{t_1^+}-1)) = \frac{(1{,}001^{360}-1)*20*25}{0{,}001}(0{,}2-(1{,}001^{33}-1)) = 35981\,€.$$

Variant 5. The seller decides whether it is necessary to establish the optimal discount to work with the compensated shortages.

$$\begin{cases} t_1^+ = \sqrt{\frac{2(R-R_1)}{r} t_k + (1+R)(t_k)^2 + t_{SW}^2} - t_k = \sqrt{2000(0{,}2-R_1)t_k + 1{,}2(t_k)^2 + 0{,}0016} - t_k \to min \\ t_k = \frac{(R-R_1)p}{K-(1+R)r_k p} = 1111{,}11(0{,}2-R_1) \end{cases},$$

$R_1 = 0{,}185$, $(1+R_1)\mathrm{p} = 23{,}7€$, $t_{20} = t_k = 18$ days, $t_1^+ = \sqrt{\frac{2(R-R_1)}{r} t_k + (1+R)(t_k)^2 + t_{SW}^2} - t_k = 32$ days ,
$t_{S0} = t_{20} + t_1^+ = 50$ days.

In 50 days it's required to bring $q_0 = \mu t_{SW}$ = 25*50 = 1250 units.
$q_{10} = \mu t_1^+$= 25*32 = 800units of product arrive at the warehouse, $q_{20} = \mu t_{20}$ = 25*18 = 450 units are sold at a price of 23,7 € on the day of delivery, a profit of 360 days will be

$$\mathrm{PR}_0 = \frac{((1+r)^T-1)p\mu}{r}(R-((1+\mathrm{r})^{t_1^+}-1)) = \frac{(1{,}001^{360}-1)*20*25}{0{,}001}(0{,}2-(1{,}001^{32}-1)) = 36122\,€.$$

Conclusions

Built economic and mathematical models of inventory management with shortages with independent of waiting time penalties let determine the best parameters of the logistic system with various combinations of restrictions for both a seller and a buyer. This leads to the increase of their adequacy and effectiveness of the practical application.

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MODELE ZARZĄDZANIA ZAPASAMI Z DEFICYTEM Z NIEZALEŻNYMI OD CZASU OCZEKIWANIA SANKCJAMI W SYSTEMIE „SPRZEDAWCA - KUPUJĄCY"

STRESZCZENIE

Parametry optymalne dla funkcjonowania systemu logistycznego sprzedawcy nie zawsze spełniają wymagania kupującego. Taka sytuacja często się pojawia podczas rozwiązywania problemów z kontroli zapasów z deficytem kompensowanym. Optymalny czas braku towaru z punktu widzenia sprzedawcy może być dość dużym. Razem z tym sankcje w przypadku czekania towaru przez kupującego nie kompensują jego straty w ciągu tego czasu. Budowanie modelu zarządzania zapasami, biorąc pod uwagę wymagania klienta, umożliwia optymalizować procesy logistyczne sprzedającego.

SŁOWA KLUCZOWE
optymalizacja, modele matematyczne i ekonomiczne, procesy logistyczne